Research Statement, Alfred Hubler, 2010

Hübler is an expert in modeling and controlling the dynamics of open dissipative systems, in particular nonlinear and chaotic oscillations and fractal particle agglomeration processes. He has been a pioneer in recent developments in complex systems research, including medium-term prediction of chaos with ensemble predictors, modeling adaptation to the edge of chaos, resonance spectroscopy with chaotic forcing functions, and modeling and prediction of the growth of fractal networks with graph theoretical methods. Hübler's underlying research goal is the development of a set of methods and paradigms to predict the dynamics and the response of complex systems.

(1) Modeling and prediction of the growth of fractal networks with graph theoretical methods

Context: Hübler carried out the first systematic experimental studies on the growth and the dynamics of fractal particle agglomerates (Dueweke, Dierker & Hübler, Phys. Rev. E **54**, 496 (1996)). His graph theoretical network analysis illustrates deficiencies in earlier fractal growth models. His work has implications for the first detailed quantitative models of fractal growth. In 1999 Hübler discovered that these self-assembling wire networks are hardware implementations of neural nets scale (Sperl, Chang, Weber & Hübler, Phys. Rev. E **59**, 3165(1999)).

Most significant recent work: Joseph K. Jun, A. Hubler, *Formation and structure of ramified charge transportation networks in an electromechanical system*, PNAS **102**, 536 (2005). This is a systematic study of the growth of fractal particle agglomerates in an electric field. The study suggests that some observables are highly reproducible, such as the number of endpoints, the number of branching points, the limiting resistance, and the fractal dimension, whereas other quantities, such as the number of trees depend sensitively on the initial condition.

Current and future work: Currently there is no known model to describe the dynamics of the observables during the growth of a fractal network. However, preliminary studies in Hübler's group suggest that a minimum spanning tree growth model might be able to reproduce the dynamics of all observables. The underlying physical equations will be used to describe certain aspects of the pattern formation, such as the opening of closed loops or the formation of linear strands at the initial stages of the growth process. The end goal is to to merge the graph theoretical and the physical model. This would allow for designing hardware implementations of neural nets both on a macroscopic scale and on a nano-scale.

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(2) Medium-term prediction of chaos with ensemble predictors

Context: Hübler introduced dynamical reconstruction methods for modeling and prediction of experimental chaotic data sets (Cremers & Hübler, Z. Naturforsch. **42**a, 797(1987)). In particular, his work on reconstructing equations of motions from experimental data with unobserved variables is often cited and widely used (Breeden, Dinkelacker & Hübler, Phys. Rev. A **42**, 5827(1990)).

Most significant recent work: C. Strelioff, A. Hubler, *Medium Term Prediction of Chaos,* Phys. Rev. Lett. **96**, 044101 (2006).

In Hübler's most recent work in chaos prediction, ensemble predictors are used to determine the trajectory of the most likely state of a chaotic system with singular points, such as a logistic map. The paper's findings make it possible to predict the behavior of a chaotic system for longer time period than with any other known prediction method. In the model, the trajectory of the most likely state stays near the images of the most likely initial state during the first time steps. Then the trajectory jumps to a new dynamics which originates at one of the singular points. Several such jumps can occur before the trajectory reaches the limiting state. The paper presents the first known evidence for these jumps and demonstrates their predictability. The paper also suggests that the trajectory of the most likely state can be much more complex and qualitatively different from the dynamics of the images of the most likely initial state.

Current and future work: The Hübler group is now experimentally reproducing the jumps in the trajectory of a chaotic system with singular endpoints. A mechanical system with chaotic motion has been completed and systematic studies are in progress.
Future work will include prediction of high-dimensional time continuous systems with low-dimensional time discrete models. For instance, if the equation of motion for the center of mass of a physical pendulum is discretized with Euler's method, the dynamics can be chaotic due to numerical instabilities. Preliminary studies suggest that this type of chaos can be found in the experiment if the time step of the discretization matches the period of the leading vibrational mode of the object. Thus, the discrete version of the equation of motion for the center of mass describes more features of the dynamics than the time continuous version. This will be studies in the context of noise in iterated functions and cellular automata, specifically, noise induced phase transitions.

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(3) Modeling adaptation to the edge of chaos

Context: Hübler was the first to recognize that seemingly erratic, random motions associated with deterministic chaos could, in fact, be controlled, and that "chaotic" systems could be steered with less effort than systems undergoing more regular motion (A. Hübler, *Adaptive Control of Chaotic Systems*, Helv.Phys.Acta **62**, 343 (1989)). Over the years, his work has inspired a large number of papers which expand his methodology or present alternative approaches.

Most significant recent work: P. Melby, J. Kaidel, N. Weber, A. Hubler, *Adaptation to the Edge of Chaos in the Self-Adjusting Logistic Map,* Phys. Rev. Lett. **84,** 5991(2000).

Hübler's paper on adapatation to the edge of chaos explores systems where chaos is suppressed due to a low-pass filtered feedback. The main finding was that a dynamical system with a range of parameter values, with periodic dynamics, and a range of parameter values, with mostly chaotic dynamics, will self-adjust toward a narrow parameter range near the boundary between the two regimes. The paper offers the first undisputed explanation for the phenomenon "adaptation to the edge of chaos", This groundbreaking phenomenon suggests that adaptive systems are much more likely to be found in a state with complex periodic dynamics or weakly chaotic motion than in a highly chaotic state or a simple periodic state. The predictions of the paper were confirmed experimentally (Melby, Weber & Hübler, Chaos **15**, 033902 (2005)) and have demonstrated a wide range of applicability (Melby, Weber & Hübler, Phys. Fluct. and Noise Lett. **2**, L285(2002)). More recently, a conserved quantity was discovered which helps to simplify models of the adaptation process (Baym & Hübler, accepted by Phys. Rev. E).

Current and future work: Current work in this area suggests that adaptation to the edge of chaos can be observed in systems with wave chaos and quantum chaos. The ongoing studies have been in the context of water-droplet shape due to wave dynamics. This system is used because the pattern of capillary waves on the surface of a vibrated water droplet depends on the shape of the droplet. Preliminary experiments show that if the droplet has an initial shape where the wave pattern is chaotic the shape changes until the wave dynamics becomes periodic. This suggests that adaptation to the edge of chaos can be observed in systems with wave chaos and quantum chaos. Future studies will include quantitative models for adaptation to the edge of chaos in spatially extended systems, non-stationary systems, and high-dimensional systems.

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(4) Resonance spectroscopy with chaotic forcing functions

Context: Hübler's work in resonance spectroscopy is based on the groundbreaking observation that nonlinear dynamical systems react most sensitive to a forcing function that complements its natural motion. In a sequence of papers he has shown that such aperiodic forcing functions have a perfect impedance match. He developed a theory of nonlinear resonance spectroscopy based on aperiodic forcing functions and his results have implications for a new generation of spectroscopic instruments with an unusually large signal to noise ratio.

Most significant recent work: V. Gintautas, G. Foster, and A. W. Hubler, *Resonant forcing of select degrees of freedom of multidimensional chaotic map dynamics.* J. Stat. Phys. 130 (3), 617-629 (2008); V. Gintautas and A. Hubler, Resonant forcing of nonlinear systems of differential equations. Chaos 18, 033118 (2008).
Hübler's 2008 papers on resonance spectroscopy explore the final response of multi-dimensional chaotic map dynamics to additive aperiodic forcing functions with equal variance. It was shown that the product of the resonant forcing function, the force function with the greatest response, and the displacement dynamics of neighboring trajectories is a conserved quantity at each time step. This is an important discovery, since it shows that the optimal forcing function is not equal but closely related to the system dynamics: it complements it. For resonance spectroscopy, the optimal forcing function is computed with a set of models, the system response reaches an absolute maximum if the model is assumed to be correct. This is a new method for system identification.

Current and future work: Physical systems, such as a chaotic coupled pendulum dynamics, are anticipated to have a conserved quantity with physical meaning, such as reaction power. Further work will explore resonances of coupled chaotic oscillators and resonances of real nonlinear oscillators with a bi-directional instantaneous coupling with a real time implementation of a model system on a computer.

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