Acceleration Beyond the Wave Speed in Dissipative Wave-Particle Systems

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The limiting speed of isotropic particles accelerated by waves is the wave speed. In the following we explore the acceleration of anisotropic objects in classical wave-particle systems. Our calculations suggest that anisotropic objects can be trapped by the waves and reach a limiting speed that is larger than the wave speed.

Gravitational and electromagnetic waves have been shown to accelerate particles to indeterminately limited relativistic velocities through resonant interaction [1]. Gyroresonant surfing acceleration [2], differing from the common wakefield model, describes particles as accelerated by a wave electric field while rotating in phase with the electric field of a circularly polarized electromagnetic wave and is a possible source of cosmic ray emergence. Renewed attempts at water wave simulation in the wake of the Sumatra-Andaman Earthquake focus on issues paralleling particle acceleration such as waves interacting with solid objects [3] and wave trapping [4], while plasma physicists conversely are studying scaling conditions for wave breaking [5] in response to promising new plasma wakefield electron accelerator developments.

Attention to plasma wakefield accelerators is exploding in answer to breaking advancements in high quality and well-populated beam production. The surfatron concept [6] emerged through the fusion of two established fields: particle acceleration and surfing. Once theoretical groundwork and viable technology was developed several paramount improvements quickly followed [7], such as the technique developed by Dawson et al. [8] showing the limitation of an electron's energy gain due to detrapping to be surpassable by imposing a magnetic field perpendicular to the plasma wave, deflecting particles along wave fronts. Recently, several concurrent breakthroughs in laser wakefield accelerators [9–12] have separately demonstrated the acceleration of particles in high-quality, well-populated beams to rates three orders of magnitude greater than conventional large-scale technology with table-top devices.

Joining two originally disparate fields, i.e. surfing and relativistic wave-particle systems, was a momentous step for describing natural phenomena such as cosmic rays and developing new technologies like plasma wakefield accelerators. Along the same thread, we propose a previously overlooked condition of particle acceleration as a candidate for new research and observing unprecedented emergent properties in particle acceleration: anisotropic wave-particle coupling.

All previous work on high-energy acceleration techniques is focussed on point-like isotropic objects with waves and time dependent fields. In this paper we investigate the high velocity acceleration of anisotropic, classical, objects with a small but finite size. We find that objects, where the viscous friction force depends on the orientation of the object, can be accelerated beyond the wave speed by planar sinusoidal waves. Anisotropic objects can be trapped by the wave if the wave amplitude exceeds a critical value. Then their limiting velocity has a non-zero component perpendicular the wave vector. We show that the limiting speed reaches a maximum for a certain orientation of the object and that objects with a certain variation of the friction constant along their principal axis approach that optimal orientation automatically. We discuss potential accelerator applications.

We consider an object with two masses m_r and m_f located at $\vec{x}_r$ and $\vec{x}_f = \vec{x}_r + L\vec{d}$. The distance L between the two masses is constant. $d = (cos\alpha, sin\alpha, 0)$ is the direction of principal axis of the object. The object is accelerated by a plane wave with wave vector, $\vec{k} = (k, 0, 0)$, amplitude A and wave speed c, where the force $F_f\vec{k} = \kappa_f A\cos(\vec{k}\vec{x}_f - kct)\vec{k}$ acts on m_f and $F_r\vec{k} = \kappa_r A\cos(\vec{k}\vec{x}_r - kct)\vec{k}$ acts on m_r in the direction of the wave vector $\vec{k}$. The equation of motion for the center of mass $\vec{x}$ is

$$m\ddot{\vec{x}} = -\mu_{\|}\vec{v}_{\|} - \mu_{\perp}\vec{v}_{\perp} + F_f\vec{k} + F_r\vec{k} \tag{1}$$

where $m = m_f + m_r$ is the mass of the object, $\vec{v_{\|}} = (\dot{\vec{x}}\cdot\vec{d})\vec{d}$ is the velocity of the object along it's principle axis. $\vec{v}_{\perp} = (\dot{\vec{x}}\cdot\vec{n})\vec{n}$ is the velocity perpendicular to the principal axis, where $\vec{n} = (-\sin\alpha, \cos\alpha, 0)$ is the corresponding normal vector. The net friction coefficients are $\mu_{\|} = \mu_{\|f} + \mu_{\|r}$ and $\mu_{\perp} = \mu_{\perp f} + \mu_{\perp r}$, where $\mu_{\|f}$ and $\mu_{\|r}$ are the friction coefficients along the principal axis for both particles and $\mu_{\perp f}$ and $\mu_{\perp r}$ are the friction coefficients perpendicular to the principal axis. For an object floating on a fluid with shallow surface waves the coupling constants are $\kappa_f = m_f g$ and $\kappa_r = m_r g$, where g is the gravitational constant.

The equation of motion for the rotation about the center of mass of the object is

$$\begin{aligned} J\ddot{\vec{\alpha}} = \mu\dot{\vec{\alpha}} + L_f\vec{d}\times\left(-\mu_{\perp f}\vec{v}_{\perp} + F_f\vec{k}\right) \\ -L_r\vec{d}\times\left(-\mu_{\perp r}\vec{v}_{\perp} + F_r\vec{k}\right) \end{aligned} \tag{2}$$

where $\vec{\alpha} = (0, 0, \alpha)$, $\mu = \mu_{\perp f}L_f^2 + \mu_{\perp r}L_r^2$, $L_f = Lm_r/m$, and $L_r = Lm_f/m$. $J = m_f L_f^2 + m_r L_r^2$ is the interia. In the following we assume that the length L is small compared to the wave length, $L \ll 2\pi/k$ and use the following approximations $\vec{F}_f + \vec{F}_r \approx (\kappa_f + \kappa_r)A\cos(\vec{x}\vec{k} - kct)$.

Numerical results generated from Eqs. (1) and (2) with a Runge-Kutta algorithm (5^{th}–6^{th} order, time step $\Delta t = 0.1$) are shown in Fig. 1. Fig. 1 shows a system where the x-velocity

v_x, approaches the wave speed c and the y-velocity v_y approaches a constant non-zero limiting value. The particle motion is trapped by the wave. The limiting speed of the object is larger than the wave speed, i.e. $v_\infty = \sqrt{v_{x,\infty}^2 + v_{y,\infty}^2} > c$, where $v_{x,\infty} = \dot{x}(t = \infty)$ and $v_{y,\infty} = \dot{y}(t = \infty)$. The limiting value of the average x-velocity is less than c, hence the particle is not trapped by the wave and the limiting speed of the object is less than the wave speed, i.e. $v_\infty < c$.

If the particle is trapped by the wave the limiting x-velocity is the wave speed: $v_{x,\infty} = c$. The x-position is $x = x_w + \Delta x$, where the distance Δx to the nearest wave front, located at $x_w = ct + n * (2\pi/k)$, approaches a constant value Δx_∞. The integer n depends on the initial conditions. The limiting distance between nearest the wavefront and the particle is:

$$\Delta x_\infty = \frac{1}{k} \arccos \frac{\mu_\| c}{\kappa A k (\cos^2 \alpha_\infty + r_\mu \sin^2 \alpha_\infty)} \tag{3}$$

where $\kappa = \kappa_f + \kappa_r$. $\alpha_\infty = \alpha(t = \infty)$ is the limiting orientation of the object and $r_\mu = \mu_\| / \mu_\perp$ is the friction anisotropy.

The argument of the inverse cosine-function $F = \mu_\| c / (\kappa A k (\cos^2 \alpha_\infty + r_\mu \sin^2 \alpha_\infty))$ in Eq. (3) is the ratio between the x-component of the friction force and the force on the object by the wave. If $F > 1$ the object de-traps. F depends on the limiting orientation α_∞. If α_∞ exceeds a critical angle, the object de-traps. The critical α-value is

$$\alpha_c = \begin{cases} 0 & \text{if } A < \frac{\mu_\| c}{\kappa k} \text{ (always de-trapped)} \\ \pi/2 & \text{if } A > \frac{\mu_\| c}{\kappa k r_\mu} \text{ (always trapped)} \\ \arcsin \sqrt{f} & \text{else (trapped if } |\alpha_\infty| \mod 2\pi < \alpha_c) \end{cases} \tag{4}$$

where $f = (1 - \frac{\mu_\| c}{\kappa A k})/(1 - r_\mu)$. The limiting y-velocity for the trapped object $v_{y,\infty}$ is dependent on the limiting orientation, α_∞, and the friction anisotropy, r_μ.

$$v_{y,\infty} = c \frac{(1 - r_\mu) \sin \alpha_\infty \cos \alpha_\infty}{cos^2 \alpha_\infty + r_\mu \sin^2 \alpha_\infty} \tag{5}$$

Fig. 2 shows the limiting y-velocity versus the orientation of the object. For a given r_μ the limiting y-velocity reaches a maximum if the limiting orientation α_∞ has the value

$$\alpha_m = \pm(\arccos \sqrt{\frac{r_\mu}{1 + r_\mu}} + i * 2\pi) \tag{6}$$

where $i = 0, 1, 2, \ldots$. The corresponding maximum limiting y-velocity is $v_y(\alpha_m) = c(1 - r_\mu)/(2\sqrt{r_\mu}$. However the object may de-trap at the optimal α-value. Hence the maximum achievable limiting y-velocity V_y for an object with friction anisotropy r_μ is

$$V_y = \begin{cases} v_{y,\infty}(\alpha_m) & \text{if } \alpha_m(r_\mu) < \alpha_c(r_\mu) \\ v_{y,\infty}(\alpha_c) & \text{else} \end{cases} \tag{7}$$

Fig. 3 shows the maximum achievable limiting y-velocity V_y versus the the friction anisotropy r_μ. For a small object ($L \ll 2\pi/k$) with coupling $\kappa_f = m_f g$, $\kappa_r = m_r g$, the object can reach V_y, if the variation of the friction along the principal axis $R = \mu_{\perp f}/\mu_\perp$ has the following value:

$$R = \frac{m_f}{m}\left(1 + \frac{m_r g A k^2 L}{\mu_\perp v_{\perp,\infty}} \sin\left(k\Delta x_\infty\right) \sin\left(2\alpha_\infty\right)\right) \tag{8}$$

where $\alpha_\infty = \alpha_m$ or $\alpha_\infty = \alpha_c$ as indicated in Eq. (7). $v_{\perp,\infty} = v_{y,\infty}\cos\alpha_\infty - c\sin\alpha_\infty$ is component of the limiting velocity in the direction of $\vec{n}$. Δx_∞ is given by Eq. (3). An object with the above R-value is accelerated to the theoretical limit of the y-velocity and reaches the largest speed possible in this system.

We have shown that classical anisotropic objects subject to viscous friction can be accelerated with planar sinusoidal waves beyond the wave speed. When the object is trapped, the component of limiting speed in the direction of the wave vector, $v_{x\infty}$ equals the phase velocity of the wave. Hence we expect that for wave packets where the phase velocity exceeds the wave speed [13], even $v_{x,\infty}$ can exceed the wave speed. In this paper we study classical objects. It should be possible to generalize these results to relativistic particles in a medium, i.e. accelerate anisotropic objects with light waves beyond the speed of light of the medium. The coupling of a charged object to polarized electromagnetic plane waves is similar to the coupling described in the paper. The relativistic wave-particle system differs in two ways, (i) the leading dissipation mechanism is radiation [14, 15] and (ii) the mass and energy of the object diverge at the speed of light in vacuum. We expect that laser-plasma beat-wave accelerator can trap certain anisotropic particles even without a large perpendicular magnetic field [8].

According to Einstein's theory of special relativity no particle can exceed the speed of light in vacuum. However, particles have been observed moving faster than the wave speed of light in media [14]. Then the leading dissipation mechanism is Cerenkov radiation. Generally, Cerenkov radiation can be described as the emission of a visible radiation with asymmetry properties dependent on the energy of the superluminal particle [15]. A cone of visible radiation appears in the medium with the apex at the point of entrance of the particle. One half of the cone formed is the angle θ and it follows the condition that $cos\theta = \frac{1}{\beta_n}$ where β is the ratio of the velocity of the particle to the velocity of light in a vacuum and n is the index of refraction of the medium for the visible light observed. The friction forces due to radiation depends on the primarily on acceleration and much less on the velocity of the object. For a velocity-dependent friction force, the mean force on the object by the wave accelerates the object to a finite limiting velocity. For acceleration-dependent friction forces this limiting velocity is infinite. Therefore we expect that in both cases the speed of an anisotropic object can exceed the wave speed. Hence it is conceivable that an anisotropcs object can be accelerated with electro-magnetic waves beyond the speed of light. In contrast, the limiting speed for isotropic objects is the speed of light.

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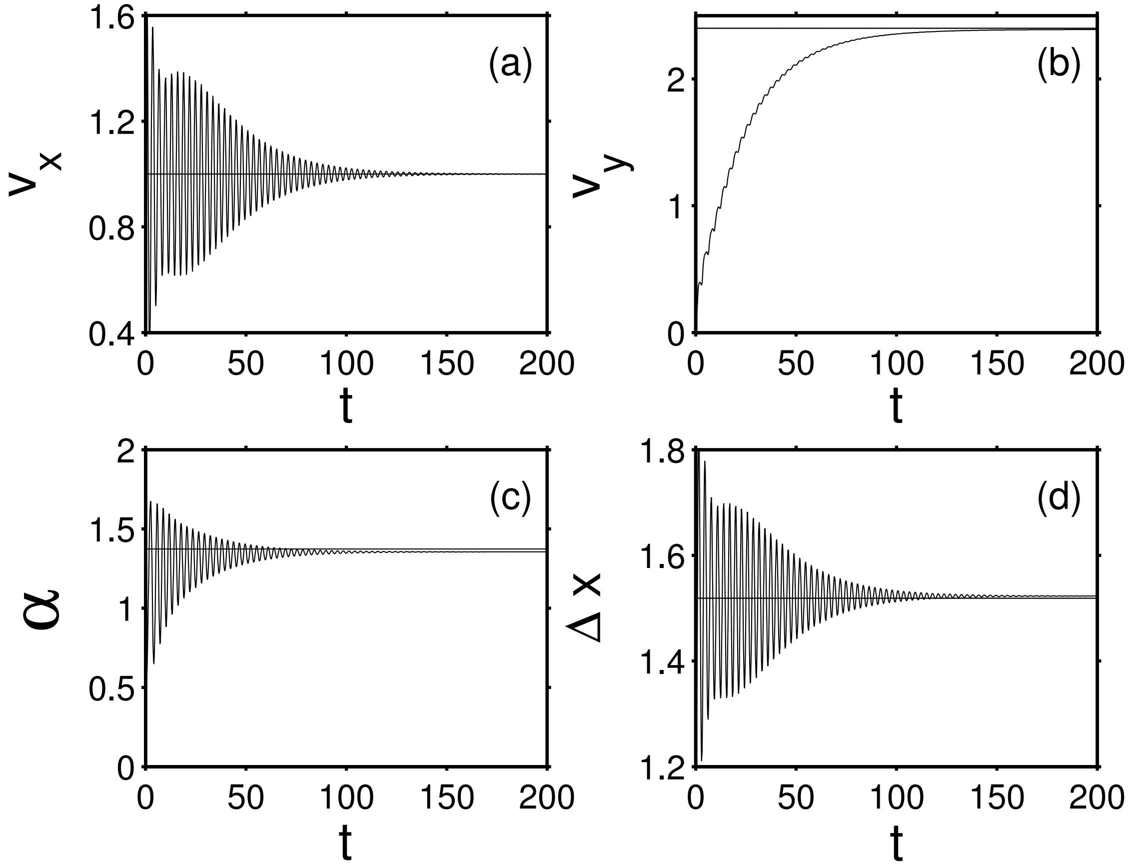

FIG. 1: Numerical simulation of a wave-particle dissipative system for the trapped case with theoretical projected values superimposed as dotted lines: x velocity, v_x, with respect to time (a); y velocity, v_y, with respect to time (b); α with respect to time (c); and the distance to the nearest wave front Δx with respect to time (d). All parameters are equal to unity except for $\mu_{\|} = 0.04$, $\mu_{\perp f} = 0.41$, $\mu_{\perp r} = 0.59$, and $L_f = L_r = 0.05$. The horizontal lines indicate the theoretical values for the limiting state of a trapped object according to Eq. 3, Eq. 5, and Eq. 8.

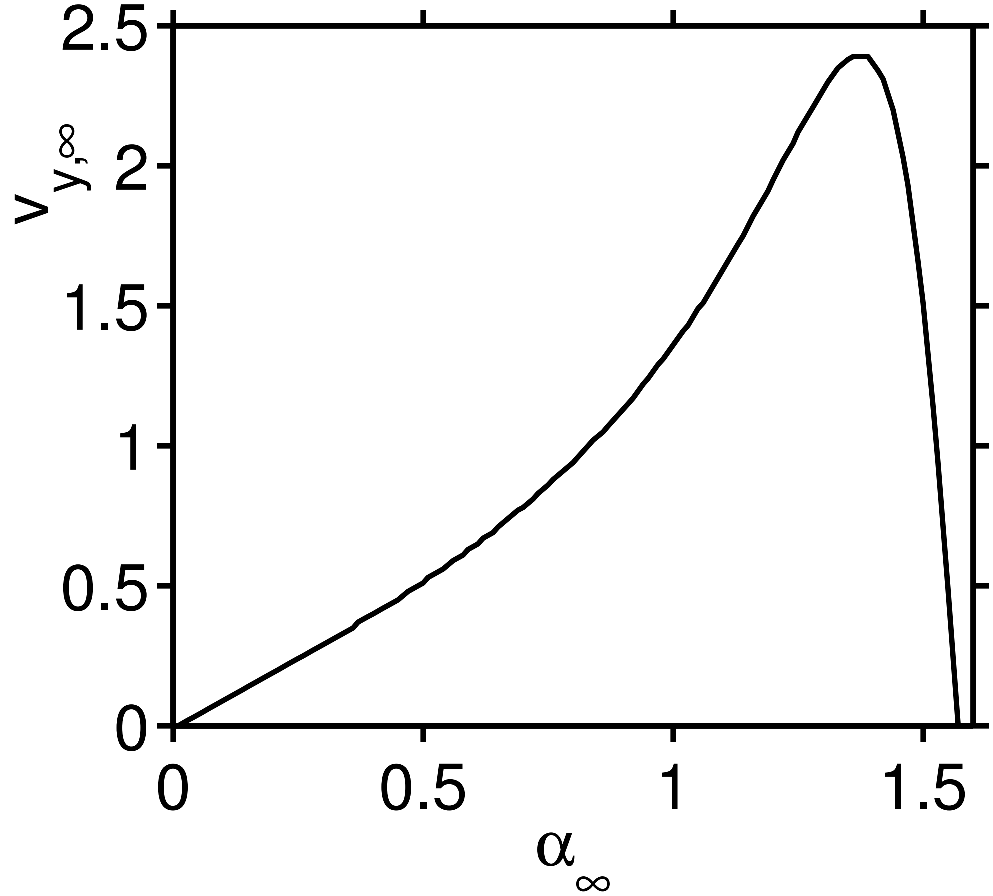

FIG. 2: The limiting y-velocity $v_{y,\infty}$ as a function of the limiting orientation α_∞ for friction anisotropy $r_\mu = 0.04$.

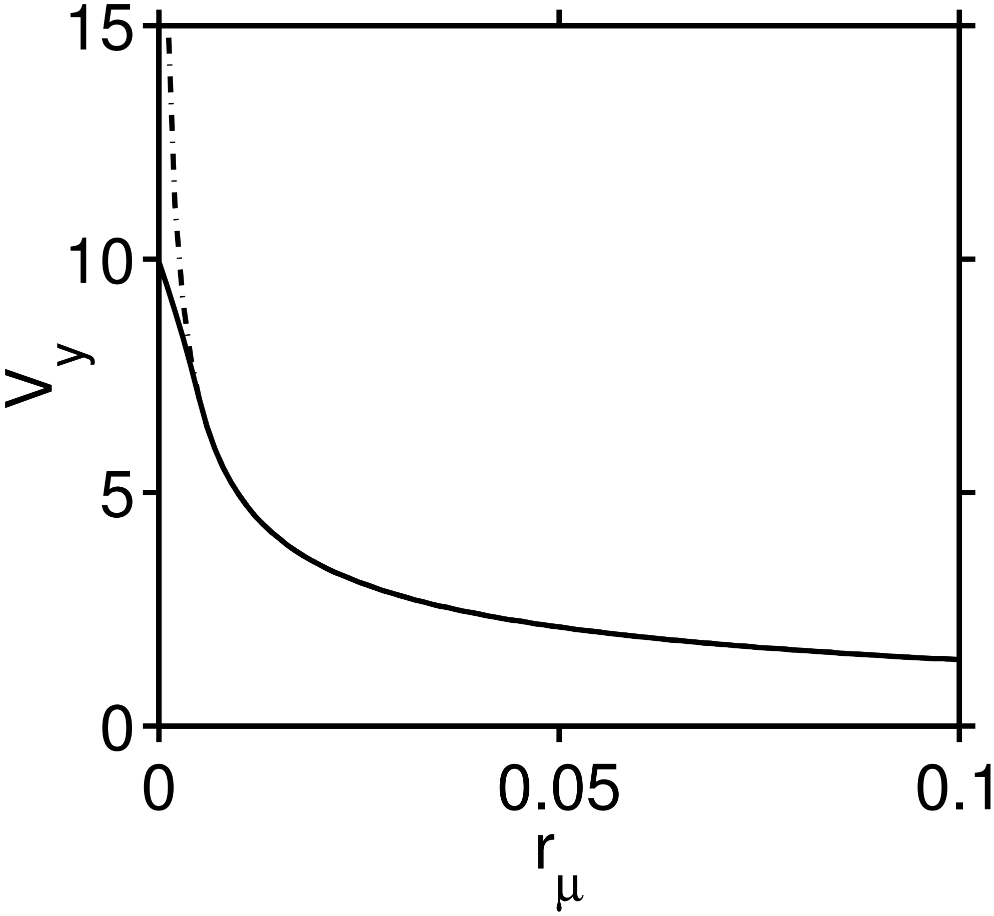

FIG. 3: The maximum achievable limiting y-velocity V_y as a function of the friction anisotropy r_μ. The dashed line shows $v_y(\alpha_m)$. All parameters are the same as in Fig. 1 except for $\mu_\parallel = 0.1$.